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SOLAR RADIATION PRESSURE ON HIGH ALTITUDE SATELLITE

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GENERAL PHYSICS RESEARCH LABORATORY

PROJECT NO. 7114

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AIR FORCE SYSTEMS COMMAND
United States Air Force

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AEROSPACE RESEARCH LABORATORIES
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This technical report presents results of research carried out by Dr. Moshe Carmeli of the Theoretical Physics Group of the General Physics Research Laboratory, Aerospace Research Laboratories, Project 7114. The research was initiated by the suggestion of Dr. Kenneth E. Kissell, Director of the General Physics Research Laboratory, in support of the Optical Properties of Space Objects (OPOS) program.

ABSTRACT

In this report an exact expression to the solar radiation pressure on a prolate spheroid is given.

INTRODUCTION

Recent studies by Fea[1], and by Fea and Smith[2], have shown the existence of an unexplained acceleration on spacecrafts which are balloon satellites of large area to mass ratio. It was suggested that the unexplained acceleration might be caused by solar radiation pressure. There is evidence[3,4] that one of the satellites showing this anomalous acceleration is no longer spherical, and that it is probably shaped like a prolate spheroid. Previously, it had been assumed that the solar radiation scattered by the satellite is symmetrical about the satellite-sun line, an assumption which no longer holds for a prolate spheroid. It is therefore expected that additional perturbations of the orbit will arise.

Consequently, Smith and Fea[5] developed a perturbation method to calculate the radiation pressure on a prolate spheroid. To this end two major assumptions were made: (1) the effective direction of reflection of the specular flux is determined by Snell's law on the incident ray that passes through the center of the satellite; and (2) that the magnitude of the flux reflected in this direction approximates to that which would be reflected by a sphere of surface area equal to that of the spheroid. Both assumptions seem not be valid in general.

In this paper we give an exact expression to the radiation pressure on a prolate spheroid without making any one of the two assumptions mentioned above.

PRELIMINARIES

It will be assumed that the solar radiation in the vicinity of the satellite is homogeneous. Also it will be assumed (see Introduction) that the satellite has the shape of a prolate spheroid. The relative orientation of the satellite with respect to the direction of light is then determined by the angle between the semi-major axis and direction of light.

We will use Cartesian coordinates defined, as usual, by $x = R \sin\theta \cos\varphi$, $y = R \sin\theta \sin\varphi$, and $z = R\cos\theta$. Let the prolate spheroid be located at the origin of coordinates and described by the equation

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{c^2} = 1. \tag{1}$$

Also, let the light rays be parallel to the x-z plane, having the direction given by (see Figure 1)

$$\vec{N} = (-\sin\theta_o, 0, -\cos\theta_o). \tag{2}$$

RADIATION PRESSURE

An infinitesimal light beam reflected from a surface element dS of the spheroid will apply a force given by

$$d\vec{F} = 2I\,(\vec{n}\cdot\vec{N})\,d\vec{S}, \tag{3}$$

where I is the light density, $\vec{n}$ is a unit vector perpendicular to the

surface element, and $\vec{N}$ is the direction of light, which is given by Eq. (2). The geometry of the surface of the prolate spheroid determines both $\vec{n}$ and $d\vec{S}$. They are given by[6]

$$\vec{n} = a\, g^{-\frac{1}{2}} \sin\theta\, (c \sin\theta \cos\varphi,\ c \sin\theta \sin\varphi,\ a \cos\theta), \tag{4}$$

$$d\vec{S} = g^{\frac{1}{2}}\, d\theta\, d\varphi\, \vec{n}, \tag{5}$$

where

$$g = a^2 \sin^2\theta\, (a^2 \cos^2\theta + c^2 \sin^2\theta). \tag{6}$$

The result is

$$d\vec{F} = -\frac{2I\, a \sin\theta\, (c \sin\theta \sin\theta_o \cos\varphi + a \cos\theta \cos\theta_o)}{(a^2 \cos^2\theta + c^2 \sin^2\theta)^{\frac{1}{2}}}$$

$$\times\, (c \sin\theta \cos\varphi,\ c \sin\theta \sin\varphi,\ a \cos\theta)\, d\theta\, d\varphi. \tag{7}$$

The total force $\vec{F}$ is, accordingly,

$$\vec{F} = \int_\sigma d\vec{F}. \tag{8}$$

Here σ stands for the surface shaded by light on which the integration should be carried out.

BOUNDARY CONDITION

The boundary condition (integration limits) of the integral of force (8) is determined by the curve C which is the boundary of the shaded area σ. This curve is given by the equation

$$\vec{N}\cdot\vec{n}(\theta,\varphi) = 0. \tag{9}$$

Using Eqs. (2) and (4) for $\vec{N}$ and $\vec{n}$ one obtains explicitly for C the equation

$$c \sin\theta_o \sin\theta \cos\varphi + a \cos\theta_o \cos\theta = 0. \tag{10}$$

This is the equation of a curve given in terms of spherical coordinates, $\varphi = \varphi(\theta)$, or

$$\varphi = \pm \text{arc cos} \left(- \frac{a}{c} \cot\theta_o \cot\theta\right). \tag{11}$$

Notice that the curve $\varphi = \varphi(\theta)$ is located in the plane

$$c \sin\theta_o\, x + a \cos\theta_o\, z = 0. \tag{12}$$

In Figure 2 we give the boundary limits where the integration on the angles θ and φ should be carried out.

TOTAL FORCE

The force integral (8) can now be given explicitly as

$$F_x = - 2I\, ac \iint \frac{\sin^2\theta\,(c \sin_o \sin\theta \cos\varphi + a \cos\theta_o \cos\theta) \cos\varphi}{(a^2 \cos^2\theta + c^2 \sin^2\theta)^{\frac{1}{2}}}\, d\varphi\, d\theta \tag{13a}$$

$$F_y = -2I\, ac \iint \frac{\sin^2\theta\,(c \sin\theta_o \sin\theta \cos\varphi + a \cos\theta_o \cos\theta) \sin\varphi}{(a^2 \cos^2\theta + c^2 \sin^2\theta)^{\frac{1}{2}}}\, d\varphi\, d\theta \tag{13b}$$

$$F_z = - 2I\, a^2 \iint \frac{\sin\theta \cos\theta\,(c \sin\theta_o \sin\theta \cos\varphi + a \cos\theta_o \cos\theta)}{(a^2 \cos^2\theta + c^2 \sin^2\theta)^{\frac{1}{2}}}\, d\varphi\, d\theta \tag{13c}$$

As can be seen from Figure 2. the limits of the double integrals in Eqs. (13) should be taken from $\varphi = -\pi$ to $\varphi = +\pi$ for $0 \le \theta \le \theta_1$, and from $\varphi = -\arccos\left(-\frac{a}{c}\cot\theta_o\cot\theta\right)$ to $\varphi = +\arccos\left(-\frac{a}{c}\cot\theta_o\cot\theta\right)$ for $\theta_1 \le \theta \le \pi - \theta_1$. The angle θ_1 is that θ for which the angle φ of the curve C satisfies $\varphi(\theta_1) = \pi$.

The force term F_y, Eq. (13b), can easily be shown to be equal to zero.

CASE OF A SPHERE

The force terms given by (13) have a particularily simple form in the case of a sphere (i.e. $a = c$). One obtains in this case

$$F_x^S = -2I\,a^2 \iint \sin^2\theta\,(\sin\theta_o \sin\theta\cos\varphi + \cos\theta_o\cos\theta)\cos\varphi\,d\varphi\,d\theta \tag{14a}$$

$$F_y^S = 0 \tag{14b}$$

$$F_z^S = -2I\,a^2 \iint \sin\theta\cos\theta\,(\sin\theta_o\sin\theta\cos\varphi + \cos\theta_o\cos\theta)\,d\varphi\,d\theta \tag{14c}$$

One can simplify these integrals by going into new variables θ', φ' in which the integration limits are not dependents on each other. To this end one rotates the coordinate system around the y-axis:

$$\begin{aligned} x &= x'\cos\theta_o + z'\sin\theta_o \\ z &= -x'\sin\theta_o + z'\cos\theta_o \end{aligned} \tag{15}$$

A simple calculation, using the relations $x = a\sin\theta\cos\varphi$, $y = a\sin\theta$

$\sin\varphi$, $z = a\cos\theta$ and $x' = a\sin\theta'\cos\varphi'$, $y' = a\sin\theta'\sin\varphi'$, $z' = a\cos\theta'$, then shows that θ and φ are related to the new variables θ' and φ' by

$$\sin\theta'\cos\varphi'\cos\theta_o + \cos\theta'\sin\theta_o = \sin\theta\cos\varphi,$$

$$-\sin\theta'\cos\varphi'\sin\theta_o + \cos\theta'\cos\theta_o = \cos\theta, \qquad (16)$$

and

$$\sin\theta' d\theta' d\varphi' = \sin\theta\, d\theta\, d\varphi. \qquad (17)$$

As a result one obtains for the force components acting on the sphere

$$F_x^S = -2I\,a^2 \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=-\pi}^{+\pi} (\sin\theta'\cos\varphi'\cos\theta_o + \cos\theta'\sin\theta_o)\cos\theta' \sin\theta'\, d\varphi'\, d\theta'$$

$$F_y^S = 0$$

$$F_z^S = -2I\,a^2 \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=-\pi}^{+\pi} (-\sin\theta'\cos\varphi'\sin\theta_o + \cos\theta'\cos\theta_o)\cos\theta' \sin\theta'\, d\varphi'\, d\theta' \qquad (18)$$

The result is

$$F_x^S = -(4/3)\pi\,a^2\,I\sin\theta_o$$

$$F_y^S = 0$$

$$F_2^S = -(4/3)\pi\,a^2\,I\cos\theta_o \qquad (19)$$

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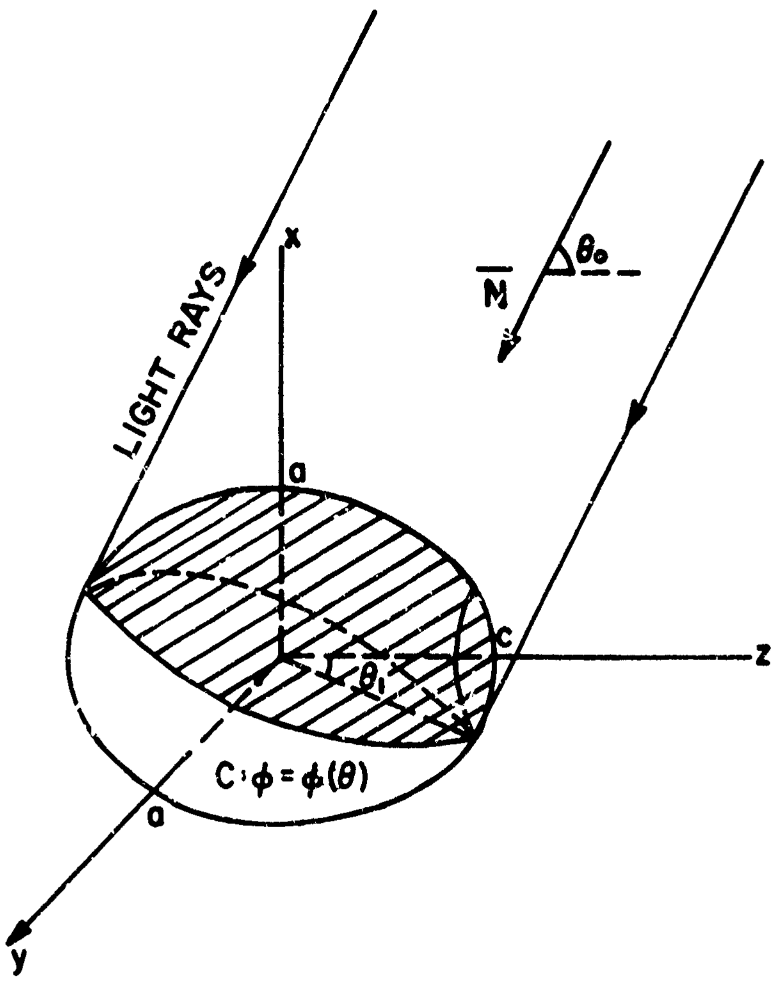

Figure 1. Prolate ellipsoid and light rays in the x-z plane along the vector $\vec{N} = (-\sin\theta_o, 0, -\cos\theta_o)$. The curve C is defined by $\vec{N}\cdot\vec{n}(\theta,\varphi)=0$, where the vector $\vec{n}$ is normal to the surface of the ellipsoid.

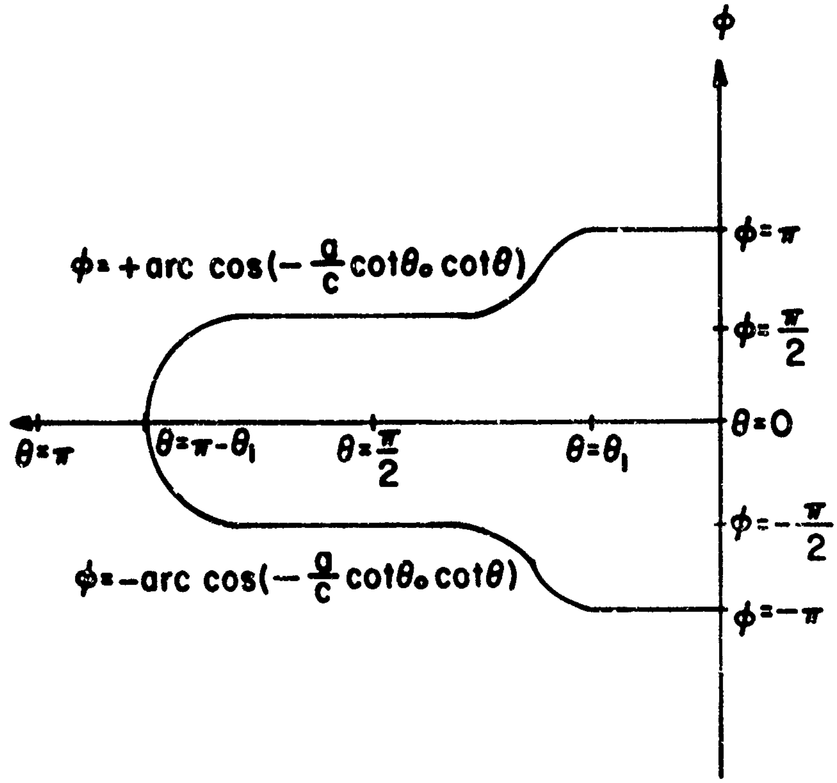

Figure 2. Boundary limit diagram. The boundary of integration is given by $\varphi = \pm\pi$ for $0 \leq \theta \leq \theta_1$, and $\varphi(\theta) = \pm$ arc $\cos(-\frac{a}{c}\cot\theta_o \cot\theta)$ for $\theta_1 \leq \theta \leq \pi-\theta_1$. The angle θ_1 is defined by θ_1 = arc tg $(\frac{a}{c}\cot\theta_o)$.

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